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Hidden Action Principal-Agent Problems with Endogenous Signal Precision

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Abstract

We study costly augmentation of signal accuracy by the principal in a binary model, with two-dimensional signal quality. Irrespective of cost, the outcomes when the principal can pre-commit to precision levels differ significantly from those when she cannot.

Keywords: information acquisition, principal agent problems, bilateral moral hazard, commitment

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1 Introduction

In principal-agent models with hidden action (see, for example, Mas-Colell, Green and Whinston (1995)), productive effort is privately costly for the agent and beneficial for the principal. While this effort is unobservable, there is a signal correlated with agent effort whose realisations are observable. The principal can design a contract, i.e., a mapping from signal realisations to transfers, to maximise her payoff, given optimal agent response. This signal generating process is typically assumed to be exogenous. In reality the principal may have scope to enhance signal quality or acquire information, possibly at some cost.[2] For example, firms or organisations often audit, inspect, supervise or monitor agents, all of which can be interpreted as activities enhancing signal precision, i.e., they reduce the stochasticity in the relationship between observation and chosen unobservable effort. Better information can enable better design of incentives and positively impact principal payoff.

This paper studies costly augmentation of signal quality by the principal, in an otherwise standard and simple binary model.[3] There are two benefit levels, high and low, with the former yielding more payoff for the principal. Higher productive effort by the agent increases the likelihood of achieving high benefit. There is a signal with two realisations, high and low, with higher benefit being more likely given a higher realisation and vice versa. Signal accuracy is completely specified by two parameters, denoting the rates of false negatives and false positives, analogous to the levels of Type I and Type II error respectively. The principal can reduce error levels by incurring cost. We study two versions, one in which she can commit to investment levels prior to offering contracts, and the other in which she cannot, and compare the outcomes to the benchmark case where signal precision levels are exogenously given.

[2] The terms 'information acquisition', 'signal quality enhancement' etc. will be used interchangeably in this article.

[3] Non-verifiable information acquisition resulting in agency problems with bilateral moral hazard has also been discussed by others (see Jost (1996), Strausz (1997), Demougin and Fluet (2001), Allgulin and Ellingsen (2002), Mitusch (2006) and Banerjee (2008)), though in contexts quite different from ours.

The two variants present sharply different results. In the former, a reduction in either error level reduces the bonus required to induce any level of productive effort. Additionally a reduction in Type II error reduces the likelihood of the bonus being given, conditional on low benefit being realised.[4] These effects can be factored in when contracts are offered, and so we find that when principal's costs become very small, the outcomes converge to those in the benchmark environment. When commitment is not possible, for a fixed bonus and level of productive effort, a reduction of Type I error only *increases* the chance, conditional on high benefit being realised, that the bonus will be paid. Hence the principal never invests in reducing Type I error. However, for a fixed bonus and level of productive effort, a reduction of Type II error *decreases* the chance, conditional on low benefit being realised, that the bonus will be paid. This difference with the full commitment scenario persists when costs are small, and we show that outcomes approach levels different from those in the benchmark case.

2 Analysis of the benchmark model

There is a principal and an agent, both risk-neutral. Each has an outside option worth 0. If the agent accepts the contract offered by the principal, he chooses effort $e \in [0, 1]$ at private cost $c(e)$. Standard assumptions are imposed on $c(e)$: $c(0) = c'(0) = 0$, $c'(x) > 0$ for $e \in (0, 1]$, $\lim_{e \to 1} c'(e) = \infty$, $c''(e) \geq 0$. The effort chosen is unobservable. The principal gets a directly captured and unobservable benefit (b), which is either $y > 0$ or 0. The agent's effort e is the probability that the principal gets benefit y.

There is a signal S correlated with the principal's benefit. The realisation of the signal is observable and can be high (s_h) or low (s_h). The conditional distribution is given below.

[4]Divergent impacts of these two errors on information acquisition incentives have been studied previously in a very different setting by Banerjee (2005).

	s_h	s_l
$b = y$	$1 - \underline{t}_1$	$\underline{t}_1$
$b = 0$	$\underline{t}_2$	$1 - \underline{t}_2$

$\underline{t}_1$ is the probability of Type I error, while $\underline{t}_2$ is the probability of Type II error, with $\underline{t}_1,\ \underline{t}_2 \in (0,\ \frac{1}{2})$. So the signal generating process has exogenous quality and the signal is informative yet imperfect. Any contract offered by the principal will therefore be defined by two terms, one giving the transfer received by the agent when $s = s_h$, which we denote z_h, and the other giving the transfer when $s = s_l$, z_l. We assume limited liability, so $z_h,\ z_l \geq 0$.

We thus have a standard principal-agent model with hidden action. Although well-known, we present the analysis below as the benchmark case, as it is also useful in the main analysis with information acquisition later. First, we make an observation which is straightforward to demonstrate and holds in this section as well as the ones below. Only a positive bonus $(z_h - z_l)$ can give the agent an incentive to put in positive effort, and agent as well as principal incentives (relevant when signal quality augmentation without commitment is discussed below) depend only on the level of the bonus. Thus we can set $z_l = 0$ without loss of generality, and view an optimal contract as only specifying a non-negative bonus z payable when s_h is realised.

Let

$$\sigma_h(e) = \Pr(s = s_h|e) = e(1 - \underline{t}_1) + (1 - e)\underline{t}_2 = \underline{t}_2 + e(1 - \underline{t}_1 - \underline{t}_2) \tag{1}$$

The expression in (1) arises as $\sigma_h(e)$ is the probability of the realisation of the high signal. If the benefit obtained is y (with probability e), s_h is realised with probability $1 - \underline{t}_1$, while the benefit obtained is 0, s_h is realised with probability $\underline{t}_2$. Hence $\sigma_h(e)$ is increasing in e. It is also decreasing in $\underline{t}_1$, and increasing in $\underline{t}_2$.

If $T(e)$ be the expected transfer from the principal to the agent, then, using (1)

$$T(e) = z\sigma_h(e) = z\underline{t}_2 + ze(1 - \underline{t}_1 - \underline{t}_2) \tag{2}$$

The second term shows that the higher is the effort, higher is the transfer received. The first term is independent of e, and is the expected transfer conditional on low benefit being obtained. Notice that the agent's marginal benefit of effort is increasing in the bonus, while given any bonus, lower values of $\underline{t}_1$ and $\underline{t}_2$ also increase marginal benefit. His choice of effort equates marginal benefit with marginal cost, so using (2)

$$z(1 - \underline{t}_1 - \underline{t}_2) = c'(e) \tag{3}$$

The principal then solves Problem $\mathcal{B}$:

$$\begin{aligned} & \max_z U = ey - z\underline{t}_2 - ze(1 - \underline{t}_1 - \underline{t}_2) \\ s.t \quad : \quad & z = \frac{c'(e)}{1 - \underline{t}_1 - \underline{t}_2} \\ \text{or equivalently} \quad : \quad & \max_e U = ey - \frac{\underline{t}_2 c'(e)}{1 - \underline{t}_1 - \underline{t}_2} - ec'(e) \end{aligned}$$

In addition to the conditions already imposed on $c(.)$, we assume throughout $c''(0) = 0$, $c'''(e) \geq 0$. These regularity conditions suffice to ensure that the first-order approach is valid and a unique interior maximum exists.[5] The first order condition is

$$y = c'(e) + ec''(e) + \frac{\underline{t}_2}{1 - \underline{t}_1 - \underline{t}_2} c''(e) \tag{4}$$

The solution is then given by (3) and (4): the principal picks the e^b which solves (4) and then sets z^b according to (3). Her payoff is U^b.

3 Precision improvement with pre-commitment

We now relax the assumption that the signal generating process is exogenous. Specifically we assume that the levels of Type I and Type II errors can be controlled by

[5] An example which satisfies all assumptions is $c(x) = \left(\frac{x}{1-x}\right)^n$, $n > 2$.

the principal. Let t_1 and t_2 be respectively be the levels of Type I and Type II error, $t_i \in [\underline{t}_i, \bar{t}_i]$, $\bar{t}_i \in (\underline{t}_i, \frac{1}{2})$, $i = 1, 2$. Assume that the cost to the principal of choosing t_i is $\beta_i k_i(t_i)$, where $\beta_i \geq 0$ is a cost parameter, and $k_i(.)$ satisfies standard assumptions: $k_i(\bar{t}_i) = k_i'(\bar{t}_i) = 0$, $k_i'(x) < 0$ for $x \in [\underline{t}_i, \bar{t}_i)$, $\lim\limits_{x \to \underline{t}_i} |k_i'(x)| = \infty$, $k_i''(x) \geq 0$. In this section we shall assume that the principal can commit to her choices when offering the contract. The rest of the model is unchanged.

We see from (2) that a higher bonus increases the agent's marginal benefit of effort, which is also increasing in the principal's investment choices. The principal then solves Problem $\mathcal{C}$:

$$\begin{aligned} & \max_{z,t_1,t_2} U = ey - \beta_1 k_1(t_1) - \beta_2 k_2(t_2) - z t_2 - ze(1 - t_1 - t_2) \\ s.t \quad : \quad & z(1 - t_1 - t_2) = c'(e) \qquad (5) \\ \text{or equivalently} \quad : \quad & \max_{e,t_1,t_2} U = ey - \beta_1 k_1(t_1) - \beta_2 k_2(t_2) - \frac{t_2 c'(e)}{1 - t_1 - t_2} - ec'(e) \end{aligned}$$

The solution $(\widehat{z}^c, \widehat{e}^c, \widehat{t_1^c}, \widehat{t_2^c})$ is interior and given by (5) together with the system

$$y = c'(e) + ec''(e) + \frac{t_2}{1 - t_1 - t_2} c''(e) \qquad (6)$$

$$\frac{t_2 c'(e)}{(1 - t_1 - t_2)^2} = -\beta_1 k_1'(t_1) \qquad (7)$$

$$\frac{(1 - t_1) c'(e)}{(1 - t_1 - t_2)^2} = -\beta_2 k_2'(t_2) \qquad (8)$$

To understand (7), observe for t_1, marginal benefit to the principal flows from the fact that a lower t_1 reduces the bonus required to induce a given e. If cost escalates (β_1 rises), the principal invests less in augmenting quality, and t_1 increases. The expression in (8) arises as for t_2 the marginal benefit flows not only because a lower t_2 reduces the bonus required to induce a given e, but additionally because a lower t_2 reduces the probability that the bonus is given, conditional on low benefit being obtained. Cost escalation leads to a rise in t_2 as well.

Now, as $\beta_1 \to 0$, $\beta_2 \to 0$, we observe from (7) and (8) that $\widehat{t}_1^c \to \underline{t}_1$, $\widehat{t}_2^c \to \underline{t}_2$. And so as $\beta_1 \to 0$, $\beta_2 \to 0$, using (3) through (6), $\widehat{z}^c \to z^c$, $\widehat{e}^c \to e^c$, $\widehat{U}^c \to U^c$ and $z^c = z^b$, $e^c = e^b$, $U^c = U^b$. Hence as the cost of improving accuracy vanishes, the principal chooses to reduce t_1 and t_2 to the minimum levels, and the optimal contract, induced effort level of the agent, and payoffs all converge to the respective values in the benchmark case.

4 Post-agreement precision augmentation

We now study information acquisition when the principal's effort choices are unobservable and undertaken after the contract is signed, keeping the remainder of the model intact.

After a contract is accepted, the agent and the principal play a game, whose Nash equilibrium outcome is anticipated when the contract is proposed. For any given contract, it is obvious that an equilibrium always exists. It is easy to show that it is unique. To see that, fix $z > 0$. Using a suitably modified version of (2), the agent's marginal payoff at e, given his conjecture $(\widetilde{t}_1, \widetilde{t}_2)$ is

$$T'(e; \widetilde{t}_1, \widetilde{t}_2, z) - c'(e) = z(1 - \widetilde{t}_1 - \widetilde{t}_2) - c'(e) \tag{9}$$

The principal chooses t_1 and t_2, given her conjecture $\widetilde{e}$. Her payoff and marginal payoffs are

$$U = \widetilde{e}y - \beta_1 k_1(t_1) - \beta_2 k_2(t_2) - T(t_1, t_2; \widetilde{e}, z)$$

$$t_1 \ : \ -T'(t_1) - \beta_1 k_1'(t_1) = z\widetilde{e} - \beta_1 k_1'(t_1) \tag{10}$$

$$t_2 \ : \ -T'(t_2) - \beta_2 k_2'(t_2) = -z(1 - \widetilde{e}) - \beta_2 k_2'(t_2) \tag{11}$$

To understand (10), recall that t_1 only affects the distribution over signal realisations conditional on high benefit being obtained. Thus, for fixed e, a lower t_1 not only increases the principal's cost, but also increases the expected transfer going to the agent, as it is now more likely that a high signal is observed, given high benefit is obtained. Overall, the marginal payoff to the principal is positive. For (11), observe that t_2 only affects the distribution over signal realisations conditional on low benefit being obtained. Hence, for fixed e, a smaller t_2 increases the principal's cost but also reduces the expected transfer going to the agent, as it is now more likely that a low signal is observed, given low benefit is obtained.

Since the net marginal benefit of t_1 is always positive, the principal chooses $t_1 = \bar{t}_1$. The equilibrium values for e and t_2 are then given by the first-order conditions derived using (9) and (11):

$$z(1-\bar{t}_1-t_2) = c'(e) \tag{12}$$

$$z(1-e) = -\beta_2 k_2'(t_2) \tag{13}$$

It is clear from (12) that the agent's best-response function is decreasing in t_2, while from (13), the principal's best-response function is increasing in e. The functions intersect only once and so the equilibrium is unique.

We now search for the optimal contract. Let $e^*(z)$ and $t_2^*(z)$ solve (12) and (13). The principal then solves Problem $\mathcal{N}$:

$$\begin{aligned} & \max_z U = ey - \beta_1 k_1(t_1) - \beta_2 k_2(t_2) - T(z; e, t_1, t_2) \\ s.t \quad : \quad & e = e^*(z), t_1 = \bar{t}_1, t_2 = t_2^*(z) \\ \text{or equivalently} \quad : \quad & \max_{e,t_2} U = ey - \beta_2 k_2(t_2) - \frac{t_2 c'(e)}{1-\bar{t}_1-t_2} - ec'(e) \end{aligned}$$

With $\widehat{t_1^n} = \bar{t}_1$, here too the solution $(\widehat{e}^n, \widehat{t_2^n})$ is interior and is given by the system

$$y = c'(e) + ec''(e) + \frac{t_2}{(1-\bar{t}_1-t_2)} c''(e) \tag{14}$$

$$\frac{(1-\bar{t}_1)c'(e)}{(1-\bar{t}_1-t_2)^2} = -\beta_2 k_2'(t_2) \tag{15}$$

We see from (14) and (15) that $\widehat{t_2^n}$ is strictly increasing in β_2, and as $\beta_2 \to 0$, $\widehat{t_2^n}$ $\to \underline{t}_2$. And so as $\beta_1 \to 0$, $\beta_2 \to 0$, the principal chooses to reduce t_2 to the minimum, while maintaining t_1 at the maximum. We have proved:

Proposition: *If the principal can commit, she always invests in reducing both types of errors. Errors levels approach their respective minima as costs go to 0. If commitment is not possible, she only invests in reducing Type II error. When costs become small, Type I error level remains at its maximum, while Type II error level approaches its minimum.*

Thus as the cost of signal quality improvement vanishes, the optimal contract, induced effort level of the agent, and payoffs approach levels different from the respective values in the benchmark situation, unlike when the principal had commitment power. The principal is better off and can induce higher effort with commitment.

5 Conclusions

We study a binary principal-agent model with hidden action where the principal can invest in improving the precision of information. When she can commit to investment choices prior to contract offer, she reduces both Type I and Type II error levels. However, if her investment choices are unobservable and cannot be undertaken before agreement, she only reduces Type II error. Hence as the costs of enhancing signal quality become vanishingly small, the outcomes in the two environments approach different values. Thus the paper demonstrates that the optimal contract in a world with free information can be very different from that in a domain with arbitrarily small costs of information acquisition. This suggests that the common notion that free and useful information is always optimally incorporated in the contract, i.e., contracting is informationally efficient in hidden-action settings, may not always easily extend when the model is perturbed by introducing endogenous information acquisition. The results of the article may be helpful in better understanding the roles of commitment power and different aspects of information quality in problems of subjective evaluation and supervision, audit, monitoring or inspection.

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