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Coordinating Monetary and Fiscal Policy for a Procyclical Real Wage

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Abstract

This paper highlights the joint role of monetary and fiscal policies in enabling procyclical movement of real wages. In the context of a monetary general equilibrium model I show that an expansionary monetary policy on its own can result in making workers and entrepreneurs better off, while an expansionary fiscal policy working in isolation will either make the worker worse off if it leads to an increase in output, or the entrepreneur worse off if it leads to a decline in output. A combination of a fiscal dole to the worker and interest-dampening monetary policy can make both entrepreneur and worker better off. This suggests that the policies of the Treasury and central bank need to be coordinated in order to bring about Pareto improvement in the economy. †

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1 Introduction

This paper highlights the joint role of monetary and fiscal policies in enabling procyclical movement of real wages. In the context of a monetary general equilibrium model I show that an expansionary monetary policy on its own can result in making workers and entrepreneurs better off, while an expansionary fiscal policy working in isolation will either make the worker worse off if it leads to an increase in output, or the entrepreneur worse off if it leads to a decline in output. A combination of a fiscal dole to the worker and interest-dampening monetary policy can make both entrepreneur and worker better off. This suggests that the policies of the Treasury and central bank need to be coordinated in order to bring about Pareto improvement in the economy.

The results of the paper rely on the assumption of diminishing marginal productivity. In the following section I outline the importance of this assumption and the manner in which it drives the main results.

2 The Role of Diminishing Productivity [1]

In an economy with perfectly competitive labor markets, and profit maximizing firms, the marginal revenue product of labor must be equal to the wage rate. If we introduce a money market from which the entrepreneur borrows money to finance production, then the marginal revenue product of labor must be equal to the price of labor times the interest rate.

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helpful comments. The usual disclaimer applies.

[1] If the production function exhibits CRS, the arguments have to be modified slightly but the same conclusions emerge.

$$MP_l = w(1 + \theta)$$

where MP_l is the marginal product of labor, w is the real wage rate, θ is the rate of interest.

With a strictly concave production function MP_l declines with an increase in output. Therefore if output increases, the real wage, can increase only if the interest rate declines by a greater proportion than the decline in MP_l. It follows that an increase in the welfare of workers is possible with an expansionary monetary policy that sufficiently reduces interest rate.

Now consider an expansionary fiscal intervention in the form of money printed and given as dole to entrepreneurs. If the intervention leads to a higher interest rate (as demand boosting intervention does in the IS-LM framework unless the economy is in a liquidity trap) then the worker cannot have a higher real wage given the profit maximizing condition of the entrepreneur. Therefore the worker must be worse off.

What if the fiscal intervention takes the form of money printed and given as dole to the worker? The rigorous consideration of this question is left for the body of the paper. Here I merely point out that as in the case of the money dole to the entrepreneur, a higher interest rate is an impediment to a higher real wage rate, given the profit maximization condition. The alleviating factor is the increased purchasing power of the worker on account of the dole. However, if the dole leads to a sufficient increase in the interest rate, then the window of opportunity opened by the dole is in some sense closed by the increase in the interest rate. Therefore the money dole and interest rate increase could "cancel each other out" leaving us with the profit maximizing condition under an interest rate which for all practical purposes is a constant. The equation now tells us that if output goes up with

interest rate constant, the real wage must decline given diminishing returns, and the worker must be worse off. In the model presented in the paper I show that an expansionary money dole always leads to a nullifying rise in interest rate of the kind discussed, thereby reducing the worker's welfare. A combination of a fiscal dole to the worker and an expansionary monetary policy could on the other hand, enhance the worker's welfare.

What of the entrepreneur's welfare? In the standard neoclassical framework with a perfectly concave production technology, the surplus of production over the wage bill increases whenever output increases. But in our model the entrepreneur is paying an interest cost beside the wage. Would the surplus still increase when the expansion is brought about by an interest-increasing fiscal dole either to the worker or the entrepreneur? In the case of a dole given to the entrepreneur, it is easy to see that if the additional purchasing power obtained through the dole is able to counteract the reduction in surplus due to the increased interest rate, then we are back in the neoclassical model with a pro-cyclical surplus. What if the dole is given to the worker? In our model, a dole given to the worker ultimately increases the purchasing power of the entrepreneur as it is used to purchase the entrepreneur's goods. Therefore again, the question boils down to whether the increase in the purchasing power through the dole is able to counteract the reduction in surplus due to the increased interest rate? I show that the money dole is always sufficient to counteract the impact of the increased interest rate, and therefore an expansion through a fiscal dole always makes the entrepreneur better off. Again, a rigorous treatment is reserved for the main body of the paper.

In order to examine these questions in a thorough manner a general equilibrium model with money is needed. However, in trying to model money in a finite general equilibrium model, one faces the Hahn Paradox which postulates that fiat money

cannot have any value in a finite horizon economy since in the last period no agent will be willing to exchange worthless paper for goods, and therefore thinking backward from the last to the first period, no agent will be willing to accept money in any period.

The paradox was addressed by Dubey-Geanakoplos(2003) through the introduction of a bank that stands ready to accept money as a repayment for loans at the end of the world, and by the assumption that sufficient gains to trade exist for people to borrow money to trade. In this paper I use a special case of the Dubey-Geanakoplos monetary economy for the analysis.

3 Model

We consider a monetary economy lasting two time periods. The economy consists of two types of households - entrepreneurs, and workers. There are two goods in the economy - labor-leisure and corn. Workers are endowed with labor in period 1 and entrepreneurs possess technology that converts labor supplied by the worker in Period 1 into corn in Period 2. This technology is given by a standard production function that satisfies the assumptions of continuity, strict concavity, impossibility of free production, and possibility of no production. Workers have preferences for corn and leisure, and entrepreneurs for corn that are characterized by utility functions satisfying the usual conditions of continuity, concavity, and monotonicity. In order to ensure existence of equilibrium we assume that the marginal product is infinite at zero units of labor input.

Money is the exclusive medium of exchange. It is fiat and gives no utility of consumption. Money enters the economy as private endowment of the entrepreneur in Period 1, and as dole given to the entrepreneur and the worker (outside money);

furthermore as money at the bank available for the long term loan(inside money). The borrowing of money occurs before commodity trade in period 1 through a long loan financial instrument. Borrowing results in the acquisition of money and entails a promise to repay the money along with the market interest at the end of period 2. Without loss of generality(as we show later) we assume that the worker is not allowed to borrow on the long loan market. The government stands ready to supply the endogenously determined quantity of bank money demanded by the households at the exogenously specified interest rate. There is no default in this model.

3.1 Government

The government has four decision variables: the rate of interest, the money supplied at the bank, the money dole to workers, and the money dole to entrepreneurs. The rate of interest and money supply are monetary variables, and the doles to workers and entrepreneurs are fiscal variables. Out of these, it can control three independently but not all four. This is the impossible quartet of the government. A choice of three of these variables is called the policy of the government. In this paper I choose the money dole to the entrepreneur, money dole to worker, and the money supply at the central bank as the policy variables and the rate of interest as the endogenous variable. The results could of course have been written with any other selection of policy variables. I carry out comparative static analysis with different policy interventions of the government.

My main objective is to examine if a fiscal boost to the economy can make both workers and entrepreneurs better off without any corresponding monetary intervention. I examine policies in which the government increases the money dole to workers without any monetary tinkering and find that if such an intervention

leads to an increase in output then the worker is worse off. Further such an intervention leads to an increase of output if and only if the entrepreneur is better off.

An isolated monetary intervention in contrast can succeed in bringing about a Pareto improvement without any accompanying fiscal adjustment.

A fiscal boost to workers accompanied by expansionary monetary policy (i.e. either increase of money supply at the bank, or a reduction in interest rates) can bring about Pareto improvement.

3.2 Sequence of Activities

3.2.1 Period 1

The following activities take place in the order listed:

(1) The long loan market meets.

(2) The labor market meets.

(3) The labor sold by the worker is employed in the production of corn.

3.2.2 Period 2

The following activities take place in the order listed:

(1) Production of corn is realized.

(2) The corn market meets.

(3) The long loan is repaid.

3.3 Notation

Let L be the labor endowment of the workers in Period 1, $u^w : R^2_+ \to R$, and $u^E : R_+ \to R$ the utility functions of the workers and entrepreneurs respectively,

$q_l^W \equiv$ quantity of labor sold by a worker in Period 1, $f : R_+ \to R$ the production function of corn, $q_c^W \equiv$ quantity of corn demanded by a worker in Period 2. The set of actions available to the worker (q_l^W, q_c^W) is denoted by q^W. Let $q_l^E \equiv$ quantity of labor demanded by an entrepreneur in Period 1, $q_c^E \equiv$ quantity of corn sold by the entrepreneur in Period 2, $q_n \equiv$ quantity of long loan bought by the entrepreneur at the beginning of Period 1[2]. The set of actions available to an entrepreneur (q_l^E, q_c^E, q_n) is denoted by q^E. Let $p_l \equiv$ price of labor, $p_c \equiv$ price of corn , $(1+\theta) \equiv$ long term interest rate. The set of prices in the economy $(p_l, p_c, 1+\theta)$ is denoted by p.

The private money endowment of the entrepreneur is m^E, the dole given to the worker is Δm^W, the dole given to the entrepreneur is Δm^E and the money at the bank available for the long term loan(inside money)is M. A policy of the government is denoted by $\wp_g$, where g is an indicator variable for the action of the government that is endogenously determined.

3.4 The Budget Set of a Household

3.4.1 Worker

We define A^W, the final allocation of a worker as follows: $A_l^W = L - q_l^W$, and $A_c^W = q_c^W$. The constraints on the set of actions q^W available to a worker given prices p are as follows:

In Period 1:

[2]The sale of q_n units of the long loan occurs before commodity trade in period 1, results in the acquisition of $\frac{q_n}{1+\theta}$ units of money and entails a promise to repay q_n units of money at the end of period 2.

Labor sold to entrepreneur $\leq$ Labor endowment

$$q_l^W \leq L \qquad (1)$$

In Period 2:

Money spent on corn $\leq$ Wage Income + Dole from government to worker

$$p_c q_c^W \leq p_l q_l^W + \Delta m^W \qquad (2)$$

The set of allocations A^W corresponding to actions q^W that satisfy these constraints is denoted by $\Sigma^W(p)$ and is called the budget set of the worker.

3.4.2 Entrepreneur

We define A^E, the final allocation of an entrepreneur as follows: $A_c^E = f(q_l^E) - q_c^E$. The constraints on the set of actions q^E available to an entrepreneur given prices p are as follows:

In Period 1: Money spent on Labor $\leq$ Money endowment + Dole from government to entrepreneur + Money obtained on long term loan

$$p_l q_l^E \leq m^E + \Delta m^E + \frac{q_n}{1+\theta} \qquad (1)$$

In Period 2: Corn sold $\leq$ corn produced

$$q_c^E \leq f(q_l^E) \qquad (2)$$

Money earned from sale of corn $\leq$ Money owed to the bank on the long term loan

$$p_c q_c^E \leq q_n \qquad (3)$$

The set of A_c^E corresponding to actions q^E that satisfy these constraints is denoted by $\Sigma^E(p)$ and is called the budget set of the entrepreneur.

4 Monetary Equilibrium

Given a policy $\wp$, a vector of allocations, prices and and endogenous government action $(A^W, A^E; p; g)$ is a monetary equilibrium if:

(1) All households are optimal on their budget sets, i.e. for workers

$$A^W \in \Sigma^W(p) \quad (1)$$

and

$$A^{\hat{W}} \in \Sigma^W(p) \Rightarrow u^W(A^{\hat{W}}) \leq u^W(A^W) \quad (2)$$

For entrepreneurs

$$A^E \in \Sigma^E(p) \quad (3)$$

and

$$A^{\hat{E}} \in \Sigma^E(p) \Rightarrow u^E(A^{\hat{E}}) \leq u^E(A^E) \quad (4)$$

(2)All markets clear, i.e. in the labor market

$$q_l^E = q_l^W \quad (5)$$

In the corn market

$$q_c^E = q_c^W \quad (6)$$

In the loan market

$$M = \frac{q_n}{1+\theta} \quad (7)$$

From the existence theorem in Dubey-Geanakoplos, for an economy with $\Delta m^W + m^E + \Delta m^E > 0$, a monetary equilibrium exists. From the proof of determinacy in Dubey-Geanakoplos(2006), equilibrium is locally unique and amenable to comparative static analysis.

4.1 Some properties of equilibrium

1. The worker will spend all the money at hand in the beginning of Period 2 on corn, else she will be left with unspent money. Thus constraint (2) of the worker's budget constraint will be satisfied with equality.

2. All the money in the system must be returned to the bank at the end of Period 2, else some household is left with unspent money. Therefore

$$1 + \theta = \frac{M + \Delta m^W + m^E + \Delta m^E}{M} > 1 \qquad (1)$$

This equation specifies the relationship between the policy variables of the government, and its fourth variable which is endogenously determined.

3. The entrepreneur spends all the money at hand after the loan market meets on purchasing labor. If not, since interest rate is strictly greater than 1, she is merely inventorying the money borrowed to pay back the bank, and in addition, having to sell for every ϵ units borrowed, $\epsilon\theta$ units worth of corn in order to repay the loan. Therefore she could be better off reducing the amount borrowed by ϵ. Therefore equation (1) of her budget constraint must be binding. This along with the market clearing condition for the labor market implies

$$p_l = \frac{M + m^E + \Delta m^E}{q_l^W} \qquad (2)$$

4. As in the case of the worker, the entrepreneur will use all the money at hand after the corn market meets in Period 2, to repay her loan, else she will be left with cash in hand. Therefore constraint (3) of her budget constraint is binding.

From point 3, the entrepreneur is not left with any money after the labor market meets. All the money in her possession after the corn market meets must have been obtained from the sale of corn. The money returned on the loan market equals $M + \Delta m^W + m^E + \Delta m^E$ and must entirely be repaid by the entrepreneur

from the proceeds of the corn sale, since the worker is not taking any loan, and the entrepreneur has no other source of repayment. Therefore the entrepreneur is receiving $M + \Delta m^W + m^E + \Delta m^E$ as payment for corn and the price of corn is given by

$$p_c = \frac{M + \Delta m^W + m^E + \Delta m^E}{q_C^W} \qquad (3)$$

5. Note that if we extended the model to allow the worker to borrow on the long loan market, at equilibrium the worker will choose not to do so. If she borrows ϵ units, she will have to return $\epsilon(1+\theta)$ to the bank. She will therefore have to withhold the money from corn purchases and sell and additional $\epsilon\theta$ worth of labor to pay back the loan. This will make her worse off.

6. The entrepreneur's profit maximization condition implies

$$MP_l \geq \frac{p_l(1+\theta)}{p_c} \qquad (4)$$

with strict equality in case of an interior equilibrium.

The following diagram depicts the equilibrium in the model. The graphs are drawn for a given policy and endogenously determined prices. As discussed, the policy determines the interest rate as per Property 2 of the equilibrium. The worker optimization panel depicts the worker's utility maximization subject to the budget constraint. The slope of the budget line is $\frac{p_l}{p_c}$ and it is shifted outward by $\frac{m^W + \triangle m^W}{p_c}$, the quantity of corn a worker can buy with their money dole. The labor-leisure panel maps the leisure consumed to the labor supplied. The labor demand panel depicts the demand for labor as a function of the interest augmented real wage $\frac{p_l(1+\theta)}{p_c}$. This is the same as the marginal product curve of the worker. The profit maximization condition sets the marginal product equal to $\frac{p_l(1+\theta)}{p_c}$. The corn supply given the equilibrium interest augmented real wage is drawn for different levels of

labor demand. The graph is determined by the budget set of the entrepreneur. Recall

$$p_l q_l^E = m^E + \Delta m^E + \frac{q_n}{1+\theta} \qquad (5)$$

from Equation (1) of the entrepreneur's budget constraints, and the point 3 of this section. Further

$$p_c q_c^E = q_n \qquad (6)$$

from Equation (3) of the entrepreneur's budget constraints, and point 4 of this section. Therefore

$$p_l q_l^E = m^E + \Delta m^E + \frac{p_c q_c^E}{1+\theta} \qquad (7)$$

From this it follows

$$q_c = \frac{p_l(1+\theta)q_l^E}{p_c} - \frac{(1+\theta)(m^E + \Delta m^E)}{p_c} \qquad (8)$$

The corn supply is translated on the y-axis and is consistent with the corn demand shown in the worker's optimization graph.

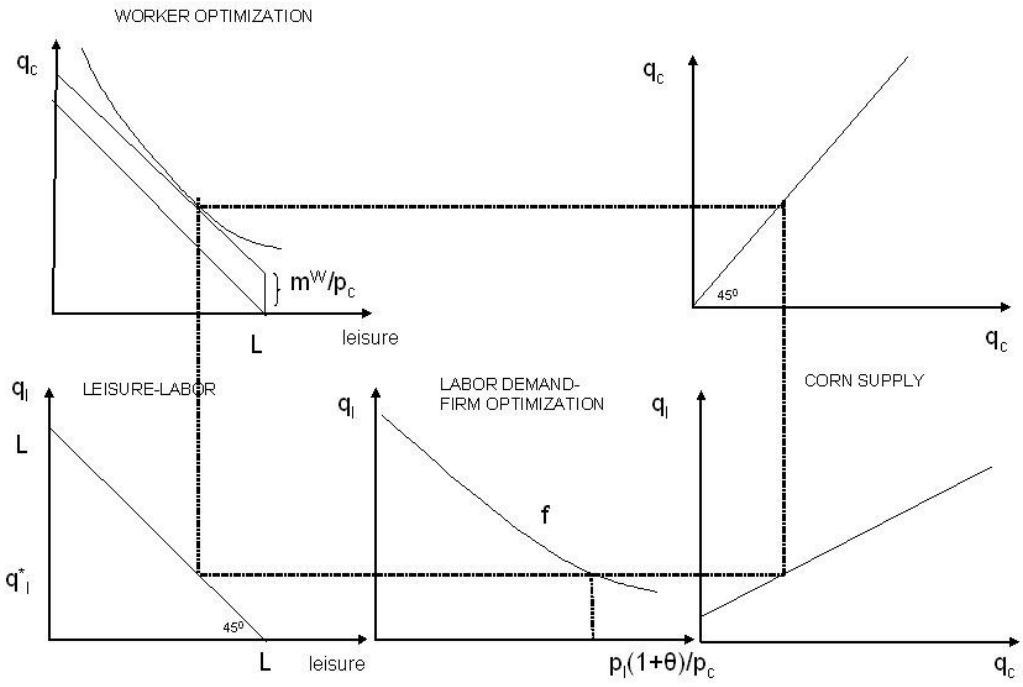

Figure 1: Equilibrium

5 Proposition and Examples

Proposition 1: *For an economy with* $\Delta m^W = 0$*, suppose government boosts demand by giving a dole to the worker* $\tilde{\Delta} m^W$ *without changing the money supply at the bank or the dole to the entrepreneur. For any pair of equilibria, one pre-intervention and one post-intervention, if the output of corn is higher in the post-intervention equilibrium, then the worker must be worse off*[3]*.*

Proof: If output of corn has gone up, the marginal product (MP_l) must have gone down as f is strictly concave. This along with equation (4) of section 4.1 implies

$$MP_l = \frac{p_l(1+\theta)}{p_c} > \tilde{MP}_l \geq \frac{\tilde{p}_l(1+\tilde{\theta})}{\tilde{p}_c} \qquad (1)$$

Note the first part of the above equation is a strict equality. By assumption the production of corn increases, therefore we are not at an interior in the pre-intervention equilibrium.

From Equation (2) of Section 4.1 the price of labor pre-intervention $p_l = \frac{M+m^E}{q_l^W}$ and post-intervention $\tilde{p}_l = \frac{M+m^E}{\tilde{q}_l^W}$.

From equation (3) of section 4.1, the price of corn pre-intervention $p_c = \frac{M+m^E}{q_c^W}$ and post-intervention $\tilde{p}_c = \frac{M+m^E+\Delta m^W}{\tilde{q}_c^W}$.

From equation (1) of section 4.1 the interest rate pre-intervention $(1+\theta) = \frac{M+m^E}{M}$ and post-intervention $1+\tilde{\theta} = \frac{M+m^E+\Delta m^W}{M}$.

Substituting the equilibrium values of prices and interest rates into (5) we get

$$\frac{q_c^W}{q_l^W} > \frac{\tilde{q}_c^W}{\tilde{q}_l^W} \qquad (2)$$

[3]The non-vacuousness of the proposition is demonstrated in Example 1.

We now complete the proof by contradiction. Suppose the worker is indifferent or better off post-intervention. Then by revealed preference his post-intervention consumption point cannot be in the interior of his pre-intervention budget set. This implies

$$p_l(\tilde{q}_l^W - q_l^W) \leq p_c(\tilde{q}_c^W - q_c^W) \qquad (3)$$

(Note, the inequality is strict in the case the worker is better off)

Substituting the equilibrium values of p_l, p_c in (3) we get

$$\frac{\tilde{q}_c^W}{\tilde{q}_l^W} \geq \frac{q_c^W}{q_l^W} \qquad (4)$$

Equation (4) contradicts equation (2). Therefore the worker must be worse off.

Q.E.D

Remark 1: Note that if output decreases it is not necessary that the worker is better off.

Remark 2: Since the reduction in welfare of the worker is driven by a diminishing productivity, it may be surmised that growth triggered by technology shocks will not make the worker worse off. However it is easy to show that an increase in output triggered by a technology shock can make the worker worse off if the number of hours have increased and the scale of the technology shock is below a certain threshold defined by the parameters of the economy. The detailed proof is presented in the Appendix.

Proposition 2: *For an economy with $\Delta m^W = 0$, suppose government boosts demand by giving a dole to the worker $\tilde{\Delta} m^W$ without changing the money supply at the bank or the dole to the entrepreneur. For any pair of equilibria, one pre-intervention and one post-intervention, the output of corn is higher in the post-*

intervention equilibrium, if and only if the entrepreneur is better off.[4]

Proof: In equilibrium the entrepreneur maximizes the difference between the corn output and the corn sold, i.e. $u^E = f(q_l^E) - q_c^E$.

From equations (1) and (3) of section 3.4.2, and points 3 and 4 of section 4.1, we get

$$q_c^E = \frac{(p_l.q_l^E - m^E)(1+\theta)}{p_c} \qquad (1)$$

From equation 4 of section 4.1,

$$MP_l = \frac{p_l(1+\theta)}{p_c} \qquad (2)$$

From equation (2) of section 4.1

$$p_l = \frac{M + m^E}{q_l^W} \qquad (3)$$

Substituting (1), (2), (3) in the expression for the entrepreneur's utility we get

$$u^E = f(q_l^E) - MP_l.q_l^E \frac{M}{M+m^E} \qquad (4)$$

Differentiating with respect to q_l^E we get

$$\frac{du^E}{dq_l^E} = MP_l \frac{m^E}{M+m^E} - \frac{dMP_l}{dq_l^E} q_l^E \frac{M}{M+m^E} \qquad (5)$$

Since f is strictly concave, $\frac{dMP_l}{dq_l^E} < 0$, and therefore $\frac{du^E}{dq_l^E} > 0 \quad \forall q_l^E$. Therefore output increases if and only if the entrepreneur is better off.

Q.E.D

[4]The non-vacuousness of the proposition is demonstrated in Example 1.

To demonstrate the proposition 1 and 2 are non-vacuous we present an example of an economy where government intervention of the type considered leads to an increase in output.

Example 1: Consider an economy with the following parameters. Workers have a utility function of the form $u^W(l,c) = min\{2l, c\}$, entrepreneurs aim to maximize their corn surplus (i.e. $f(q_l^E) - q_c^E$), the production function is of the form $f(l) = 6.2l - .4(l)^2$ with free disposal for all $l > 7.75$. The workers are endowed with 10 units of labor and the entrepreneurs with one unit of money. The government fixes interest rate $1 + \theta$ at $\frac{11}{10}$.

The equations of equilibrium for an equilibrium in which the equilibrium labor supply is less than 7.75 are as follows:

$$2q_l^\star = q_c^\star \quad Utility\ maximization$$

$$6.2 - 0.8q_l^\star = \frac{p_l^\star(1+\theta)}{p_c^\star} \quad Profit\ maximization$$

$$p_l^\star q_l^\star = p_c^\star q_c^\star \quad Worker's\ budget\ constraint$$

$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad Entrepreneur's\ budget\ constraint$$

$$p_c^\star q_c^\star = 11 \quad Loan\ market\ clears$$

The market clearing conditions for the labor and corn markets are implied in the equations above by setting $q_l^W = q_l^E = q_l^\star$ and $q_c^W = q_c^E = q_c^\star$.

The following is an equilibrium in this economy: Entrepreneurs borrow \$ 10 from the bank by issuing promissory notes of \$ 11, 5 units of labor are exchanged in Period 1 at \$ 2.2 a unit, and 10 units of corn are exchanged in Period 2 at \$ 1.1 a unit. Workers consume 5 units of leisure and 10 units of corn, entrepreneurs have a corn surplus of 11 units.

Suppose the government gives a dole of $ 1 to the worker without changing the money dole to the entrepreneur or the money supply. Then $1 + \theta$ rises to 1.2.

The equations of equilibrium for an equilibrium in which the equilibrium labor supply is less than 7.75 are as follows:

$$2q_l^\star = q_c^\star \quad \textit{Utility maximization}$$

$$6.2 - 0.8q_l^\star = \frac{p_l^\star(1+\theta)}{p_c^\star} \quad \textit{Profit maximization}$$

$$p_l^\star q_l^\star + 1 = p_c^\star q_c^\star \quad \textit{Worker's budget constraint}$$

$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad \textit{Entrepreneur's budget constraint}$$

$$p_c^\star q_c^\star = 12 \quad \textit{Loan market clears}$$

As before, the market clearing conditions for the labor and corn markets are implied in the equations above.

The following is an equilibrium in the post-intervention economy at which output has gone up, the entrepreneur is better off and the worker is worse off:

Entrepreneurs borrow $ 10 from the bank by issuing promissory notes of $ 12, 5.5 units of labor are exchanged in Period 1 at $ 2 a unit, and 9 units of corn are exchanged in Period 2 at $ $\frac{4}{3}$ a unit. Workers consume 4.5 units of leisure and 9 units of corn, entrepreneurs have a corn surplus of 13 units.

Notice that in the example presented, the worker's share of the total corn produced has declined. It can easily be shown that whenever the output of corn goes up, the share of the worker in corn consumption, and indeed, the absolute amount of corn consumed by the worker, must decline.

An expansionary fiscal intervention that takes the form of a money dole to the entrepreneur with a simultaneous hike in interest rate to clear the money market

will also lead to a decline in welfare of the worker and an increase in welfare of the entrepreneur.

6 Pareto Improvement with An Isolated Monetary Intervention

Propositions 1 and 2 indicate the challenge in improving the welfare of the worker with a fiscal intervention not accompanied by a monetary adjustment. However as suggested in the introduction, a monetary intervention that reduces the interest rate might fare better. We demonstrate that this is indeed the case through an example. Consider a policy intervention under which the government increases the stock of money at the bank and does not tinker with any fiscal variable. The interest rate which is determined endogenously will fall. It is possible that both worker and entrepreneur are better off with output increase, provided the percentage fall in the rate of interest is higher than the percentage fall in the marginal product.

Example 2: Consider an economy with the following parameters. Workers have a utility function of the form $u^W(l,c) = min\{\frac{c}{42} + \frac{205}{21}, l + \frac{c}{2}\}$, entrepreneurs aim to maximize their corn surplus (i.e. $f(q_l^E) - q_c^E$), the production function is of the form $f(l) = 2.2l$. The workers are endowed with 10 units of labor and the entrepreneurs with one unit of money. The government gives a money dole of \$ 1 to the workers and fixes interest rate $1+\theta$ at $\frac{12}{10}$.

The equations of equilibrium are as follows:

$$q_l^\star + \frac{10}{21} q_c^\star = \frac{205}{21} \quad Utility\ maximization$$

$$2.2 = \frac{p_l^\star (1+\theta)}{p_c^\star} \quad Profit\ maximization$$

$$p_l^\star q_l^\star + 1 = p_c^\star q_c^\star \quad Worker's\ budget\ constraint$$
$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad Entrepreneur's\ budget\ constraint$$
$$p_c^\star q_c^\star = 12 \quad Loan\ market\ clears$$

The following is an equilibrium in this economy: Entrepreneurs borrow $ 10 from the bank by issuing promissory notes of $ 12, 5 units of labor are exchanged in Period 1 at $ 2.2 a unit, and 10 units of corn are exchanged in Period 2 at $ 1.2 a unit. Workers consume 5 units of leisure and 10 units of corn, entrepreneurs have a corn surplus of 1 unit.

Suppose the government increases M to 11 to balance the loan market. It continues to give a dole of $ 1 to the worker.

The equations of equilibrium are as follows:

$$q_l^\star + \frac{10}{21} q_c^\star = \frac{205}{21} \quad Utility\ maximization$$
$$2.2 = \frac{p_l^\star (1+\theta)}{p_c^\star} \quad Profit\ maximization$$
$$p_l^\star q_l^\star + 1 = p_c^\star q_c^\star \quad Worker's\ budget\ constraint$$
$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad Entrepreneur's\ budget\ constraint$$
$$p_c^\star q_c^\star = 13 \quad Loan\ market\ clears$$

The following is an equilibrium in the post-intervention economy at which interest rate has gone down, output has gone up, and both the entrepreneur and worker are better off.

Entrepreneurs borrow $ 11 from the bank by issuing promissory notes of $ 13, 6 units of labor are exchanged in Period 1 at $ 2 a unit, and 12.1 units of corn are exchanged in Period 2 at $ $\frac{130}{121}$ a unit. Workers consume 4 units of leisure and 12.1 units of corn, entrepreneurs have a corn surplus of 1.1 unit.

7 Pareto Improvement with Coordinated Monetary and Fiscal Intervention

If interest rate were fixed at zero then the Dubey-Geanakoplos economy is identical to the Arrow-Debreu economy. By the First Welfare Theorem the equilibria would be Pareto-optimal. Therefore in the previous section, it was the wedge of the positive interest rate that drove the equilibrium away from efficiency and allowed us to bring about a Pareto improvement.

In this section we demonstrate the possibility of a Pareto improvement through a policy intervention that leads to an *increase* in the interest rate, i.e. amplifies the wedge that drove the economy away from efficiency in the first place. Of course by Theorem 1, if the interest rate rise is not dampened by expansionary monetary policy the worker cannot be better off. So the trick is to let the fiscal intervention lead to rise in the interest rate but dampen it slightly with a monetary expansion.

Example 3: Consider an economy with the following parameters. Workers have a utility function of the form $u^W(l,c) = min\{\frac{c}{63} + \frac{205}{21}, l + \frac{31c}{63}\}$, entrepreneurs aim to maximize their corn surplus (i.e. $f(q_l^E) - q_c^E$), the production function is of the form

$$f(l) = \{ \begin{array}{l} 2.2l, \forall l \leq 5.9 \\ \frac{121l}{57}, otherwise \end{array}$$

The workers are endowed with 10 units of labor and the entrepreneurs with one unit of money. The government fixes interest rate $1+\theta$ at $\frac{11}{10}$.

The equations of equilibrium for an equilibrium in which the equilibrium labor supply is less than 5.9 are as follows:

$$q_l^\star + \frac{30}{63} q_c^\star = \frac{205}{21} \quad Utility\ maximization$$

$$2.2 = \frac{p_l^\star(1+\theta)}{p_c^\star} \quad Profit\ maximization$$

$$p_l^\star q_l^\star + 1 = p_c^\star q_c^\star \quad Worker's\ budget\ constraint$$

$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad Entrepreneur's\ budget\ constraint$$

$$p_c^\star q_c^\star = 11 \quad Loan\ market\ clears$$

The following is an equilibrium in this economy: Entrepreneurs borrow $ 10 from the bank by issuing promissory notes of $ 11, 5 units of labor are exchanged in Period 1 at $ 2.2 a unit, and 10 units of corn are exchanged in Period 2 at $ 1.1 a unit. Workers consume 5 units of leisure and 10 units of corn, entrepreneurs have a corn surplus of 1 unit.

Suppose the government increases the dole to the workers $\triangle m^W$ from 1 to 2, and increases M to 19 while keeping the dole to entrepreneurs constant.

The equations of equilibrium for an equilibrium in which the equilibrium labor supply is greater than 5.9 are as follows:

$$q_l^\star + \frac{30}{63} q_c^\star = \frac{205}{21} \quad Utility\ maximization$$

$$\frac{121}{57} = \frac{p_l^\star(1+\theta)}{p_c^\star} \quad Profit\ maximization$$

$$p_l^\star q_l^\star + 2 = p_c^\star q_c^\star \quad Worker's\ budget\ constraint$$

$$p_l^\star q_l^\star = \frac{p_c^\star q_c^\star}{1+\theta} + 1 \quad Entrepreneur's\ budget\ constraint$$

$$p_c^\star q_c^\star = 22 \quad Loan\ market\ clears$$

The following is an equilibrium in the post-intervention economy at which output has gone up, and both the entrepreneur and worker are better off.

Entrepreneurs borrow $ 19 from the bank by issuing promissory notes of $ 22, 6 units of labor are exchanged in Period 1 at $ $\frac{10}{3}$ a unit, and 12.1 units of corn

are exchanged in Period 2 at \$ $\frac{20}{11}$ a unit. Workers consume 4 units of leisure and 12.1 units of corn, entrepreneurs have a corn surplus of a little over 1.09 units.

One may surmise that in the manner of the example above it would be possible to construct an example in which a dole to the entrepreneur with a simultaneous dampening of the interest rate results in a Pareto improvement. The following proposition shows that this is in fact, impossible.

Proposition 3: *For an economy with* $\Delta m^W = 0$*, suppose government boosts demand by giving a dole to the entrepreneur* $\tilde{\Delta} m^E$ *and placing an additional amount of money at the bank to clear the loan market at the pre-intervention interest rate. For any pair of equilibria, one pre-intervention and one post-intervention, if the output of corn is higher in the post-intervention equilibrium, then (a) the worker must be worse off and (b)the entrepreneur must be better off .*

Proof: (a) The proof is identical to the proof for Proposition 1 since we can treat $\frac{M}{M+m^E}$ as a constant.

(b) Identical to the proof for proposition 2.

Q.E.D

8 Concluding Remarks

The paper presents a simple model to highlight the powerful role of interest rate movements in determining whether real wages move pro-cyclically or countercyclically. It points out that a fiscal dole targeted to workers must always be accompanied by a monetary expansion in order to have the desired effect. The coordination of the Treasury and the central bank is therefore essential.

9 References

[1] Dubey, Pradeep , Geanakoplos, John - Inside and Outside Fiat Money, Gains to Trade, and IS-LM, Economic Theory, 2003, 21, (2-3), pp. 347-397.

[2] Dubey, Pradeep , Geanakoplos, John - Determinacy with nominal assets and outside money, Economic Theory, 2006, Vol 27, No 1, pp 79-106.

10 Appendix: Expansion Caused by Technology Shock

Proposition: *Consider an increase in output and employment triggered by a technology shock that increases productivity by a factor of k. If $k < \overline{\alpha}$ then the worker must be worse off.*

Proof: Consider a production function f and a productivity enhanced production function $\tilde{f}$ such that $\tilde{f}(.) = kf(.)$.

From the concavity of f it follows that for every increase in employment from q_l^E to $\tilde{q}_l^E$, there exists an upper bound $\overline{\alpha}$ such that if $k < \overline{\alpha}$, then

$$\frac{d\tilde{f}}{dq_l^E}(\tilde{q}_l^E) = k\frac{df}{dq_l^E}(\tilde{q}_l^E) < \frac{df}{dq_l^E}(q_l^E)$$

$$\Rightarrow \tilde{MP}_l(\tilde{q}_l^E) < MP_l(q_l^E)$$

By the earlier argument for the entrepreneur's FOC (see proof of Proposition 1, equation 6), this implies

$$\frac{\tilde{q}_c^W}{\tilde{q}_l^W} < \frac{q_c^W}{q_l^W} \qquad (1)$$

But if the worker is better off after the productivity increase, by the worker's

revealed preference, as in proof of Proposition 1, equation 8

$$\frac{\tilde{q}_c^W}{\tilde{q}_l^W} \geq \frac{q_c^W}{q_l^W} \qquad (2)$$

This contradicts equation(1). **Q.E.D**

Example 4: Consider an economy with the following parameters:

$$u^W(l^W, c^W) = min\{l^W + \frac{5c^W}{6}, \frac{240l^W}{171} + \frac{108c^W}{171}\}$$

$$u^E(c^E) = ln\ c^E$$

$$f(q_l^W) = \begin{cases} 6.2q_l^W - .4(q_l^W)^2, \forall q_l^W \leq 7.75 \\ 24.025, \forall q_l^W > 7.75 \end{cases}$$

$$L = 10; m^W = 0; m^E = 1; 1 + \theta = \frac{11}{10}$$

The following is an equilibrium in this economy:

$$q_n = 11; q_l^W = q_l^E = 5.5; q_c^E = q_c^W = 9$$

$$p_l = 2; p_c = \frac{11}{9};$$

Suppose productivity scales up by a factor of $\frac{22}{21}$ so that $\tilde{f}(q_l^W) = \frac{22}{21} f(q_l^W) \forall q_l^W$. The following is an equilibrium in the post-technology shock economy at which output and employment have increased, the entrepreneur is better off and the worker is worse off:

$$q_n = 11; q_l^W = q_l^E = 6; q_c^E = q_c^W = 8$$

$$p_l = \frac{11}{6}; p_c = \frac{11}{8}$$