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Multiple Equilibria in a Simple Model of Search with Entry

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Abstract

Diamond (1971) analyzed the following goods market: identical prospective buyers with unitary demand searched sequentially over identical monopolistically competitive firms. The equilibrium market price was shown to be unique and equal to the monopoly price. Suppose, now, that in order to participate in a Diamond-style goods market, prospective buyers are charged a small but positive entry fee. Since the ex post market price extracts all the consumer surplus from entering buyers, no one finds it worthwhile to pay the entry fee (i.e., the goods market shuts down). In order to study the non-trivial implications of consumer entry, I modify the Diamond-model slightly.

[1]This paper has greatly benefited from discussions with Professor Peter Diamond. Of course, the usual disclaimer applies.
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The modified model displays two interesting features - buyers with positive entry fees enter the goods market, and the goods market generates multiple equilibrium prices.

1 Introduction

In a seminal paper, Diamond (1971) analyzed a goods market with the following features: identical prospective buyers with unitary demand and gross valuation, v, searched sequentially, at a fixed cost per observation, over identical monopolistically competitive firms. A remarkable result was established: regardless of the magnitude of the search cost, the equilibrium market price was unique and equal to the monopoly price, v. Since real-world retail markets are frequently characterized by a substantial degree of price variation, Diamond's "unique market price" conclusion posed a perplexing paradox.

Motivated in part by the above paradox, several authors have constructed market models that exhibit price dispersion in equilibrium. I mention here a few of the important contributions. Considering models of monopolistic competition, Albrecht and Axell (1984), Axell (1977), Rob (1985), Salop and Stiglitz (1982), and Stiglitz (1987) obtain price dispersion when buyers differ in search costs; Diamond (1987) emphasizes the case wherein buyers have different valuations of the good; while Reinganum (1982) explores a setup in which firms have different production costs. In models of non-sequential search, Butters (1977), Burdett and Judd (1983), and Salop and Stiglitz (1977) demonstrate that price dispersion emerges when buyers differ ex post in the number of price offers received. Finally, Shilony (1977) and Varian (1980) analyze models of oligopoly and identify price dispersion with the mixed strategy pricing behavior of firms.

In this note, I explore yet another aspect of the Diamond (1971) model; specifically, I consider the issue of market entry by buyers. Assume first that in order to participate in a Diamond-style goods market, prospective buyers are charged a small but positive entry fee. Since the ex post equilibrium market price extracts all the consumer surplus from entering buyers, no one finds it worthwhile to pay the entry fee. In sum, with a positive entry fee, trading in the goods market ceases.

To study the non-trivial implications of buyer entry, certain modifications in the Diamond (1971) structure are clearly required. I examine a model that differs from Diamond's setup in two essential ways. First, I assume that there is heterogeneity in the buyer pool with regard to the magnitude of the entry fee. Furthermore, for some buyers (e.g., teenagers in a shopping mall or hagglers in a bazaar), the entry fee is posited to be negative. Second, I impose heterogeneity in the buyer pool with regard to the gross valuation of the commodity.

My model generates two interesting propositions. First, I establish that in equilibrium, some buyers, though subjected to a strictly positive entry fee, nonetheless enter the goods market. Thus, the model highlights an externality. Buyers with negative entry fees enter the goods market regardless of the ensuing market price. Their participation makes it more likely that buyers with positive entry fees choose to trade as well. Second, the model generates multiple steady-state equilibrium prices. The multiple equilibria can be ranked in terms of aggregate welfare: the trade volume and aggregate welfare decrease as the equilibrium price is raised.

The rest of this note is structured as follows. In section 2, I describe the search model and analyze its equilibrium. In section 3, I conclude by discussing the implications of the model's solution.

2 The Search Model

I consider a market for a homogeneous commodity with agents of two sorts: buyers, the potential demanders of the good, and firms, the suppliers of the good. Time is measured in discrete intervals and the horizon is infinite.

Buyer Attributes. In generic period t, buyers of unit measure are born. A buyer survives for two periods; thus, birth in period t leads to death at the conclusion of period $(t+1)$.

After birth, each generation-t buyer decides whether to enter the period-t goods market. Should a buyer opt to enter, she is subjected to an entry fee, denoted by c. In each generation, a fixed proportion $\mu \in (0,1)$ of buyers have c-values that are negative. Such buyers receive positive benefits from market participation per se. For the remaining buyers, of proportion $(1-\mu)$, a more conventional assumption is invoked. Their c-values are represented by a cumulative probability distribution function $F(c)$ with density $f(c) > 0$ defined on the interval $[0, C]$ ($F(0) = 0$ and $F(C) = 1$).

Upon entering the goods market, a generation-t buyer has the option to purchase one unit of the commodity in either period t or period $(t+1)$. Consumption of the commodity yields an instantaneous benefit, denoted by v. Different buyers have different v-values. In each generation of buyers, the v-values are represented by a cumulative probability distribution function $G(v)$ with density $g(v) > 0$ defined on the interval $[0, C]$ ($G(0) = 0$ and $G(C) = 1$). For analytical tractability, c and v are assumed to be independent random variables.

Consider a buyer born in period t. Associated with this buyer is her pair of attributes (c, v). Four cases need to be reviewed. First, should the buyer opt for non-entry in the period-t goods market, her net payoff is 0. Second, if the buyer enters the period-t goods market and purchases the commodity in period t at price p_t, her net payoff is $(v - p_t - c)$. Third, if the buyer enters the period-t goods market and purchases the commodity in period $(t+1)$ at price p_{t+1}, her net payoff is $[\delta \times (v - p_{t+1}) - c]$, where $\delta \in (0, 1)$ is the discount factor common to all buyers. Fourth, should the buyer enter the period-t goods market and exit without purchase, her net payoff is $-c$.

Firm Attributes. There are m infinitely-lived firms in the goods market. Consider the actions of firm i in period t. At the start of period t, on a take-it-or-leave-it basis, firm i sets a period-t price, denoted by p_{it}. Thereafter, by a process to be described shortly, buyers are assigned to firm i. Given p_{it}, each assigned buyer decides whether to procure the good from firm i: firm i's period-t demand equals the number of purchasing buyers.

In setting price p_{it}, firm i's objective is to maximize its discounted profit stream. I shall impose assumptions to ensure that firm i's dynamic problem reduces to a static one. For notational ease, I also set firm i's cost of production to 0.

Matching of Consumers to Firms. At the start of period t, buyers in the goods market are of two sorts. In the first category are buyers born in period $(t-1)$ who entered the goods market in period $(t-1)$, rejected the price offered in period $(t-1)$, but decided nonetheless to sample the period-t price. In the second category are buyers born in period t who enter the period-t goods market. In period t, each buyer (from both the above categories) is randomly assigned to one of the m firms in the goods market. Since a firm's allotment of buyers is independent of its price history, the optimally chosen p_{it} maximizes firm i's profits in period t.

Entry of Consumers. I shall restrict attention to equilibria with an additional property:

$$p_{it} = p, \forall i, t \tag{1}$$

Thus, in equilibrium, a time- and firm-independent price, p, prevails in the goods market. All buyers correctly anticipate p. Hence, a generation-t buyer enters the goods market if her (c, v)-pair satisfies 1) $c < 0$, or 2) $c \geq 0$ and $(v - p - c) \geq 0$.

Aggregate Demand in Equilibrium. I now derive the period-t aggregate demand at price p. How many generation-t buyers purchase the commodity at price p? First, a proportion μ of generation-t buyers have c-values that are negative. These buyers enter the period-t goods market and purchase the commodity immediately (that is, in period t) if gross valuation, v, weakly exceeds p. Hence, aggregate demand from such buyers equals $[\mu \times (1 - G(p))]$.

A proportion $(1-\mu)$ of generation-t buyers have non-negative c-values. For this group, a buyer with (c, v)-pair such that $(v-p-c) \geq 0$ enters the period-t market and purchases the commodity immediately (that is, in period t). Hence, aggregate demand from such buyers equals $[(1-\mu)\times\int_p^C F(v-p)dG(v)]$.

Finally, observe that in period t, the generation-$(t-1)$ buyers still in the market are those with $c < 0$ and $v < p$. Such buyers do not contribute to the period-t aggregate demand at price p. In sum, the period-t aggregate demand at price p, denoted $D(p)$, is as follows:

$$D(p) = [\mu \times (1 - G(p))] + [(1 - \mu) \times \int_p^C F(v - p)dG(v)] \tag{2}$$

An Extra Assumption. Consider a generation-t buyer with the following two characteristics: 1) $c < 0$ and 2) $v < p$. Since $c < 0$, the buyer enters the period-t goods market. Since $v < p$, in equilibrium the buyer does not procure the commodity in period t or period $(t + 1)$. I shall maintain that such a buyer exits the goods market after one round of search with a time-invariant probability of $(1 - \pi) \in (0, 1)$.

The description of the search model is now complete. Proposition 1 establishes the *existence* of an equilibrium market price in the open interval $(0, C)$. Proposition 2 demonstrates that the model generates *multiple* market price equilibria.

Proposition 1. *For the search market, the existence of a time- and firm-independent equilibrium price in the open interval* $(0, C)$ *is guaranteed.*

Proof: The solution consists of two parts. Given a putative equilibrium price p, I first ensure that it is unprofitable for firm i in period t to raise its price from p to $(p+\epsilon)$. Thereafter, I verify that it is also unprofitable for firm i in period t to lower its price from p to $(p - \epsilon)$.

Raising price to $(p + \epsilon)$. Suppose firm i charges a price of $(p + \epsilon)$. I evaluate the firm's period-t demand, denoted $D_i(p+\epsilon)$, in two steps. First, all generation-$(t-1)$ buyers assigned to firm i in period t have gross valuations, v, less than p. Hence, they do not contribute to demand $D_i(p + \epsilon)$. Second, a generation-t buyer assigned to firm i in period t behaves as follows: 1) if gross valuation, v, is such that $(v - p - \epsilon) \geq \delta \times (v - p)$, the buyer purchases the good from firm i; otherwise 2) the buyer exits firm i and purchases the good in period $(t + 1)$ at the putative equilibrium price of p.

I now compute the number of generation-t buyers assigned to firm i in period t with v-values satisfying $(v - p - \epsilon) \geq \delta \times (v - p)$ - that is, $v \geq k \equiv [p + \frac{\epsilon}{1-\delta}]$. Two distinct situations arise. First, a mass $\frac{\mu}{m}$ of generation-t buyers with firm i have negative c-values. Of these buyers, a proportion equal to $[1 - G(k)]$ have gross valuations weakly exceeding k. Second, generation-t buyers of mass $(1 - \mu)$ have c-values that are non-negative. Since p is the conjectured market price, the proportion of such buyers that 1) enter the period-t goods market and 2) have gross valuations weakly exceeding k equals $[\int_k^C F(v-p)dG(v)]$. Given the random matching process, firm i obtains a $\frac{1}{m}$ share of all entering buyers. Thus, $D_i(p+\epsilon)$ is as follows:

$$D_i(p+\epsilon) = \frac{1}{m} \times [\mu \times (1 - G(k)] + \frac{1}{m} \times [(1-\mu) \times \int_k^C F(v-p)dG(v)] \quad (3)$$

When firm i sets a price of $(p+\epsilon)$, its period-t profits, denoted $R_i(p+\epsilon)$, is $(p+\epsilon) \times D_i(p+\epsilon)$. If "market price of p" is an equilibrium, $R_i(.)$ must decline for local increases of price relative to p. This is equivalent to the following condition:[2]

$$D_i(p) \leq \frac{p \times g(p) \times \mu}{m \times (1-\delta)} \quad (4)$$

Lowering price to $(p - \epsilon)$. Suppose firm i charges a price of $(p - \epsilon)$. I evaluate firm i's period-t demand, denoted $D_i(p - \epsilon)$, in two steps. First, firm i is assigned generation-$(t-1)$ buyers of mass $\frac{\mu \times \pi \times G(p)}{m}$. All such buyers have gross valuations v less than p (otherwise, purchase and exit would have occurred in period $(t-1)$ itself). The proportion of such buyers with $v \in [p - \epsilon, p)$ is given by $\frac{[G(p) - G(p-\epsilon)]}{G(p)}$. Since these buyers procure the good from

[2] I will assume that the second-order condition for profit maximization is satisfied. The second-order condition is satisfied if 1) $g'(v) \geq 0$, or 2) $g(v) \geq \frac{|g'(v)|}{1-\delta}$. Note that if the v-values are uniformly distributed among buyers, then $g'(v) = 0$. Hence, in this case both conditions 1 and 2 hold.

firm i at the reduced price $(p-\epsilon)$, their contribution to $D_i(p-\epsilon)$ equals $\frac{\mu\times\pi\times[G(p)-G(p-\epsilon)]}{m}$.

Consider the behavior of a generation-t buyer assigned to firm i in period t. The buyer purchase the good from firm i if and only if her gross valuation v weakly exceeds $(p-\epsilon)$. How many such buyers arrive? First, a mass $\frac{\mu}{m}$ of generation-t buyers with firm i have c-values that are negative. Of these buyers, a proportion equal to $[1-G(p-\epsilon)]$ have gross valuations weakly exceeding $(p-\epsilon)$. Second, a mass $(1-\mu)$ of generation-t buyers have non-negative c-values. Since p is the conjectured market price, the proportion of such buyers that 1) enter the period-t goods market and 2) have gross valuations weakly exceeding $(p-\epsilon)$ equals $[\int_p^C F(v-p)dG(v)]$. Given the random matching process, firm i receives its $\frac{1}{m}$ share of all entering buyers. Thus, $D_i(p-\epsilon)$ is as follows:

$$D_i(p-\epsilon) = \frac{\mu\times\pi\times[G(p)-G(p-\epsilon)]}{m} + \frac{\mu\times\ [1-G(p-\epsilon)]}{m}$$

$$+\frac{(1-\mu)\times\int_p^C F(v-p)dG(v)}{m} \quad (5)$$

When firm i sets a price of $(p-\epsilon)$, its period-t profits, denoted $R_i(p-\epsilon)$, is $(p-\epsilon)\times D_i(p-\epsilon)$. If "market price of p" is an equilibrium, $R_i(.)$ must decline for local decreases of price relative to p. This is equivalent to the following condition:[3]

$$D_i(p) \geq \frac{p\times g(p)\times\mu\times(1+\pi)}{m} \quad (6)$$

[3]I will assume that the second-order condition for profit maximization is satisfied. The second-order condition is satisfied if 1) $g'(v)\leq 0$, or 2) $g(v)\geq\frac{|g'(v)|\times v}{2}$. Note that if the v-values are uniformly distributed among buyers, then $g'(v)=0$. Hence, in this case both conditions 1 and 2 hold.

Using equations (4) and (6), a market price of p can be sustained in equilibrium if and only if the corresponding aggregate demand (refer to equation (2)) satisfies the following condition:

$$\frac{p \times g(p) \times \mu}{1-\delta} \geq D(p) \geq p \times g(p) \times \mu \times (1+\pi) \tag{7}$$

I have assumed that $g(p)$ is a continuous function and strictly positive on the interval $[0, C]$. At $p = 0, D(p)$ strictly exceeds 0 whereas at $p = C, 0 = D(C) < [C \times g(C) \times \mu \times (1+\pi)]$. Since $D(p)$ is a continuous function, the Intermediate Value Theorem guarantees that the set $S_L \equiv \{p | D(p) = [p \times g(p) \times \mu \times (1+\pi)]; p \in (0, C)\}$ is non-empty. Given that $\pi \in [0, 1]$ is arbitrary, I shall choose π such that $(1+\pi)$ is strictly less than $\frac{1}{1-\delta}$. Then, by construction, $\frac{p \times g(p) \times \mu}{1-\delta}$ strictly exceeds $p \times g(p) \times \mu \times (1+\pi)$. As a result, every element of S_L satisfies equation (7). In sum, I have proved the existence of a steady-state equilibrium price in the open interval $(0, C)$.

Proposition 2. *For the search market, there exist multiple time- and firm-independent equilibrium prices.*[4]

Proof: Note that at $p = 0, D(p)$ strictly exceeds 0 whereas at $p = C, 0 = D(C) < \frac{C \times g(C) \times \mu}{1-\delta}$. Therefore, the Intermediate Value Theorem guarantees that the set $S_R \equiv \{p | D(p) = \frac{p \times g(p) \times \mu}{1-\delta}; p \in (0, C)\}$ is non-empty. Furthermore, for $\pi \in [0, 1]$ chosen such that $(1+\pi)$ is strictly less than $\frac{1}{1-\delta}$, it is clear that every element of S_R constitutes a steady-state equilibrium price. Observe, finally, that S_L and S_R are disjoint sets. Hence, I have proved the existence

[4]Notice that I have only established the *existence* of multiple steady-state equilibria. In many cases - for example, when $g(.)$ and $f(.)$ are densities corresponding to the uniform distribution - there is a continuum of steady-state equilibrium prices. I have not emphasized this aspect since it is tangential to the main thrust of this note.

of multiple steady-state equilibria.

3 Conclusion

In this note, I have constructed a model of search with two features - *buyers with positive entry fees enter the goods market, and the goods market generates multiple equilibrium prices*. My construction uses two critical assumptions. First, I have introduced heterogeneity among prospective buyers with regard to gross valuations, v. Suppose, instead, that all prospective buyers share a common gross valuation, equal to v_c. Standard arguments establish that the unique market price in this case is also v_c. In equilibrium, only buyers with negative c-values enter the goods market and purchase the commodity. Therefore, without heterogeneity in gross valuations, my model collapses to that of Diamond (1971). Second, I have maintained that a strictly positive fraction, μ, of prospective buyers have negative entry fees. Suppose, instead, that $\mu = 0$. Then a direct application of the arguments in this note (details omitted for brevity) reveals that the unique market price is C. In other words, with $\mu = 0$, the goods market shuts down.

I now focus on a particular equilibrium market price, say $\hat{p}$. Exclusively because of the search environment (note the absence of strategic interactions in my model), each firm's demand curve has a kink at $\hat{p}$. Why? Suppose firm i in period t raises its price from $\hat{p}$ to $(\hat{p}+\epsilon)$. Then it loses generation-t buyers assigned to it with gross valuations v satisfying $\delta \times (v - \hat{p}) > (v - \hat{p} - \epsilon)$. Such buyers respond to the price hike by searching for one more period; the good is purchased in period $(t + 1)$ at price $\hat{p}$. Note that firm i's demand loss is increasing in δ. Now, suppose firm i in period t lowers its price from

$\hat{p}$ to $(\hat{p} - \epsilon)$. Given the structure of my model, the lower price does not induce additional search from prospective buyers assigned to other firms. Firm i's increase in demand comes from generation-$(t-1)$ buyers assigned to it in period t with gross valuations v satisfying $v \geq (\hat{p} - \epsilon)$. Clearly, firm i's demand gain is increasing in π, the time-invariant probability with which generation-$(t-1)$ buyers remain in the period-t goods market. When $(1+\pi) < \frac{1}{1-\delta}$ (refer to condition (7)), the elasticity of demand with respect to a price increase relative to $\hat{p}$ exceeds the elasticity of demand with respect to a price decrease relative to $\hat{p}$. This kink, in turn, sustains $\hat{p}$ as an equilibrium market price.[5]

The central result of this note is the multiplicity of equilibrium market prices. In other words, the location of the kink in a firm's demand curve is indeterminate. What accounts for this indeterminacy? When prospective buyers conjecture a low market price, even those with low gross valuations enter the goods market. Given the resulting distribution of gross valuations in the goods market, individual firms find it optimal to charge a low price, thereby validating buyers' initial beliefs. By contrast, when prospective buyers predict a high market price, only those possessing high gross valuations enter the goods market. With high-type buyers populating the goods market, individual firms discover that charging a high price maximizes private profits. Hence, firms' pricing behavior matches buyers' forecasts.

The multiplicity of equilibrium market prices has an interesting impli-

[5]Stiglitz (1987) also uses a search environment to show that a firm's demand curve may have a kink at the market price. My model, which is undoubtedly less involved, differs from Stiglitz (1987) in several ways - Stiglitz's buyers demand $x(p)$ units of the good at price p; when a firm changes its price from the putative equilibrium, the altered price distribution is *immediately* known to all prospective buyers; etc.

cation. Following convention, I measure aggregate welfare as the sum of consumer and producer surplus. Given the structure of the model, it is immediate that aggregate welfare increases as the volume of sales rises. Thus, the model's multiple equilibria are ranked in terms of aggregate welfare: aggregate welfare decreases as the equilibrium price is raised.

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