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Equilibrium: Statics and Dynamics

In Memory of
Gerard Debreu and Michio Morishima

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Introduction

The year 2004 saw the passing away of two giants among economic theorists: Michio Morishima and Gerard Debreu. Morishima died on July 13, 2004 at the age of 80 in a hospital in England. Gerard Debreu died in France at the

¹It is impossible to do justice in a few pages to the work of stalwarts such as Debreu and Morishima; it was felt that an attempt should be made to record our indebtedness to their work. I am indebted to Takao Fujimoto, Satish Jain and Tapas Majumdar for very helpful comments and suggestions; I am particularly grateful to Takao Fujimoto for help in removing a factual inaccuracy.

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age of 83 on December 31, 2004. Both Debreu and Morishima spent much of their lives as expatriates, in countries that they were not born in: Morishima, a Japanese, spent a fairly long and productive part of his later life in the U.K., teaching at the University of Essex and the LSE; Debreu, French by birth, spent his entire working life in the U.S., first with the Cowles Foundation at Chicago and then Yale, and finally, most of his later professional life at UC Berkeley; ultimately, he retired to spend the last years in France. Both of course were scholars of the first rank. Debreu spent his life studying general equilibrium theory: in fact a website of the UC Berkeley, mentions his Field as Mathematical Economics, his Past Research Topics: Theory of General Equilibrium and his Current Research Topics: Theory of General Equilibrium. This single minded approach and focused attention to a general area is very rare indeed. Morishima was interested in problems of general equilibrium theory too but his focus was on problems of dynamics, growth and development and lately in the fate of the Japanese Economy. We present below our appreciation of their works and contribution.

Gerard Debreu

The year was 1975 and the occasion was the Third World Congress of the Econometric Society and the venue was the University of Toronto in Canada. It was also my first major conference². A very special high point of the conference was that I got to listen to the presentation by Gerard Debreu. I clearly recall having reached the hall slightly in advance; the hall was empty

²This conference will always be special for me for many reasons. The fact that the book of abstracts distributed at the Conference had a very special first page, first item, is not the only reason.

except for a single person who was busily cleaning the huge blackboard³. After cleaning the blackboard, the person turned around and came up to me and said: “Well, I seem to be guaranteed of an audience.”. I was really thunderstruck; since I had never seen Debreu before, I had not recognized him. I mumbled something and he must have realized that I had been caught unawares. He asked me what I did and once he learned that I taught at JNU, in New Delhi, he asked me about the flight and then asked me where I had done a Ph.D. from and then went on to ask me about my dissertation. I had by then relaxed sufficiently to be able to converse somewhat easily; by then people had started coming in to the lecture hall and he kept on nodding and waving but he kept talking to me, about my work and the paper I had presented at the conference. Then it was time for him to get back to the podium for his lecture and he went back with a friendly wave. I remember some one asking me what we had been talking about and I could cockily say that “Gerard seemed to like what I presented”. I didn’t let on about the fact that I had known Professor Debreu for all of 5 minutes or so. His lecture, if I recall correctly, was on regular differentiable economies.

Having been a student of Lionel McKenzie, one had been exposed to a fair bit of Debreu. I recall of course the first time that I started to read Debreu’s classic “Theory of Value” (TOV)⁴: that was immediately after my M.A. in Kolkata, I used to sit at some lectures that Professor Tapas Majumdar used to give for research students and the interested outsiders⁵. In fact in my first

³OHP, powerpoint etc. had still not made an appearance and people still used blackboards!

⁴G. Debreu, (1959), **Theory of Value**, Wiley, New York.

⁵I belonged to the latter category at that time.

year as a graduate student, the graduate student advisor⁶ had advised that I should take the course taught by McKenzie rather than the Introduction to Mathematical Economics course but that I should take Professor McKenzie's permission. When McKenzie learnt that I had read Value and Capital and Debreu's Theory of Value, he said "Maybe I cannot teach you anything new then and you don't need to do my course".

Gerard Debreu's work on General Equilibrium Theory set standards which became benchmarks; that these benchmarks could hardly be met is quite another matter. He was among the first to introduce the axiomatic method; in fact a subtitle of his classic book is "An Axiomatic Analysis of Economic Equilibrium"; this method involves the selection of primitive concepts of analysis: the commodity space, the price system, the households with characteristics like preferences, endowments and profit shares, firms with their technology or production possibility sets. Next, each of these were represented explicitly by a mathematical object; for instance, the commodity space is a linear space, the price system is a linear functional on the commodity space, the preferences are binary relations on the space and so on. The properties of these mathematical entities were made explicit next: the commodity set is a subset of a finite dimensional real vector space; the number of households is finite, binary relations are defined to be continuous, complete and transitive and so on. Implications of these assumptions are drawn next: these are the results or theorems of this axiomatic method.

The method has several advantages: what is being said is clearly defined; assumptions are clearly stated and conclusions are rigorously established. The only unjustified claim, that I can think of, in the celebrated TOV is the

⁶The late Sherwin Rosen. He was an excellent graduate student advisor.

claim that one does not need to have a mathematical background: Chapter 1, first paragraph says, “Its reading requires, in principle, *no* knowledge of mathematics”. However, Debreu did go on to qualify this claim by saying that this was true in principle only. “It requires an ability to think abstractly, which is usually developed through the practice of mathematics, and an ability to assimilate in a short time and even an elementary knowledge of the integers are taken for granted.” The level of abstraction that Debreu demands of course makes matters difficult. For to quote Bertrand Russell, “many people have a passionate hatred of abstraction, chiefly, I think, because of its intellectual difficulty; but as they do not wish to give this reason, they invent all sorts of others that sound grand. They say that all abstraction is falsification...”⁷. In defense of this method, it is best to quote Debreu himself, “ The recent effort towards axiomatization of economic theory therefore seems to be fully supported by Francis Bacon’s assertion in the *Novum Organum*, citius emergit veritas ex errore quam ex confusione or that truth emerges sooner from error than from confusion.”⁸. The main areas of his contributions were welfare economics, existence of equilibrium, large economies and the core, utility and demand theory and regular differentiable economies. Ignoring his most well known contribution jointly made with Arrow, on the existence of a competitive equilibrium⁹, I shall choose a few results for special

⁷Russell, B., (1931), **The Scientific Outlook**, Allen and Unwin, London., quoted in Hahn’s Inaugural lecture, *On the Notion of Equilibrium in Economics*, (1974), Cambridge University Press, Cambridge.

⁸See, for instance Hildenbrand, W., 1983, Introduction, in **Mathematical Economics, Twenty Papers by Gerard Debreu**, Cambridge University Press, Cambridge. This Introduction contains a very clear evaluation of the work of Debreu.

⁹On this aspect, the interested reader is referred not only to TOV mentioned above but to Debreu’s own masterly survey in the 1982 Volume II of the **Handbook of Mathemat-**

mention to provide a flavor of his work.

The first result relates to the conditions required for continuous representation of preferences, or the existence of a continuous real valued utility function. As any teacher of economics, particularly microeconomics, will testify, introducing the notion of a utility function is often greeted with complete disbelief. However the basic idea that in order to choose, consumers must have a ranking over all admissible alternatives, is easily acceptable. The passage from the ranking to a continuous utility function is entirely due to Debreu; he identified a set of conditions on the rankings, which would allow the existence of such a function: Debreu, always very careful, of course referred to this function as an utility indicator function. This method allows one to conclude that there may be some rankings which are not amenable to such representation¹⁰.

The second result which I wish to mention is in the paper he wrote jointly with Scarf on the Core¹¹. One of the features of a competitive equilibrium which is most commonly talked about is the relationship between Competitive Equilibria and Pareto Optima. The set of results in this connection are often referred to as the Fundamental Theorems of Welfare Economics: Debreu shed light on this relationship as well in some of the basic foundational papers on the subject. But there is another aspect of a competitive outcome which was noted by Edgeworth who made the conjecture that if one wishes to

ical Economics, Arrow, K. J. and M. D. Intrilligator eds., North Holland, Amsterdam.

¹⁰G. Debreu, (1954), Representation of a Preference Ordering by a Numerical function, in **Decision Processes**, R. M. Thrall, C. H. Coombs and R. L. Davis eds., Wiley, New York.

¹¹G. Debreu and H. Scarf (1963), A Limit Theorem on the Core of the Economy, **International Economic Review**.

distribute goods among individuals so that ensuing distribution would be readily acceptable by all, then a competitive solution will fit the bill. Ready acceptability means that no individual or groups of individuals can suggest an alternative distribution amongst its members, using their own resources, such that no one is worse off and some one is actually better off. Debreu and Scarf showed, borrowing a solution concept from cooperative game theory, the core, that the core shrinks to the set of competitive equilibria, as the number of participants increases. This essentially proved the Edgeworth Conjecture. The construction and the results are splendid examples of ingenuity; the passage to infinitely many participants was achieved by keeping the number of types of individuals fixed while allowing the common number of participants in each type to become large. More general methods were achieved later but this set the trend and allowed the literature on Large Economies to develop¹².

The next example I have chosen relates to the number of equilibria. Not being satisfied with showing existence, Debreu must have realized that what had to be done was to establish the conditions under which equilibrium was unique. Once again, a classic Debreu paper established the fact that for most economies, the number of equilibria was finite¹³; this paper was among the pioneers in applying measure theory. There have been several refinements. Debreu was never really interested in employing strong conditions to achieve a given result: it would appear that his quest was for generality and simplicity and he withdrew whenever he realized that unwarranted assumptions needed to be employed. Thus Debreu said “ economies with multiple equilibria must be allowed for. Such economies seem to provide a satisfactory explanation

¹²See, for instance, W. Hildenbrand, (1974), **Core and Equilibria for Large Economies**, Princeton University Press, Princeton.

¹³G. Debreu, (1970), Economies with a Finite set of Equilibria, **Econometrica**.

of equilibrium as well as a satisfactory foundation for the study of stability provided that all the equilibria are locally unique. But if the set of equilibria is compact (a common situation), local uniqueness is equivalent to finiteness.”¹⁴

For any economic model, there are three consistency checks that need to be made¹⁵ : the first is naturally the existence of a configuration where all participants can in fact carry out their objectives: the existence of an equilibrium. The second check is whether this configuration is unique and the third and final check is whether the equilibrium is stable in some sense. I have already mentioned the contributions of Debreu towards the first two checks. So far as the third is concerned, in a remarkable and clear contribution, which laid bare the foundation of ongoing research in this connection, Debreu¹⁶ showed that in the context of a competitive equilibrium, the properties of Walras Law, homogeneity of degree zero in the prices and continuity in prices are not enough to imply any non-trivial restriction on an economy; and by implication, to obtain stability we must be compelled to employ additional, unwarranted assumptions¹⁷.

In 1983, The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel (the so-called Nobel prize in Economics) was awarded to Gerard Debreu “for having incorporated new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium”. It is said that when some one called up Debreu to congratulate him

¹⁴See the first paragraph of the 1970 *Econometrica* paper referred to above.

¹⁵See for example, A. Mukherji, (2002), *Introduction to General equilibrium Analysis*, **Oxford University Press**, Chapter 4.

¹⁶G. Debreu, (1974), *Excess Demand Functions*, **Journal of Mathematical Economics**.

¹⁷See, for example, Hildenbrand’s *Introduction* referred to above.

for the award, he brushed it aside thinking that some one was making fun of him.

To sum up, competitive equilibrium theory has been subjected to very close scrutiny by a person who pursued the truth in steadfast and single-minded manner. He contributed greatly to the growth and establishment of the Department of Economics at the University of California, Berkeley, produced many students who are leaders in the field and set standards for rigour for economic theorists all over the world.

Michio Morishima

My first introduction to Michio Morishima was in 1967 or so, when I was thinking of applying for graduate work in economics abroad. Given that I did not have a Master's degree in Economics, prospects were not very bright. One of my former teachers in Presidency College in Kolkata, advised that I should consider applying to two places: University of Essex in Colchester, UK and University of Rochester, New York, USA. His advice was based on the fact that Professor Morishima was in Essex then and Professor McKenzie was in Rochester, and given my background in mathematics, I may be found acceptable at these places. It turned out that he was correct in this and I was offered admission to both of these places; Essex was first but sadly they did not have any fellowship to offer to graduate students. Rochester came next with a full fellowship and tuition waiver. Naturally the choice was clear and I wrote to Essex saying that I had decided to accept the Rochester offer. I received a very encouraging letter from Professor Morishima stating that I was doing the right thing and that he was sure that given the well known

Rochester graduate program, I would prosper. I had the privilege of meeting him in person in 1977 when I had been appointed to a Lectureship at the London School of Economics and Political Science (LSE). I was informed that I would have to begin with 15 lectures on Imperfect Competition and that I should get in touch with Michio Morishima who had been giving these lectures and wanted to be relieved of this responsibility. In addition, LSE, then had the practice of associating each lecturer with a Professor as caretaker, and Morishima had been requested to look after me. I wrote to him requesting some information about what kinds of things he discussed during these lectures. It may be recalled that in the mid seventies, imperfect competition was not the vast area we know today. The major books were by Joan Robinson and Edward Chamberlin and this had been so for the previous 40 years or so. James Friedman's first book had just come out and some papers were being written in journals. But this set of 15 lectures were for undergraduates doing mathematical economics and no adequate material was really available. Morishima sent me his lecture notes and these helped me to set the level of my own lectures. On reaching the LSE, I asked Morishima for permission to distribute his notes to my students and he immediately said "certainly not" with great force and went on to say that I should prepare my own notes. This provided me with the impetus to prepare my own notes on the subject. I was to find out that Professor Morishima being my caretaker meant that I had to meet him from time to time. The meeting had to be scheduled which meant that I had to ask the lady who was assigned to be my secretary (she was actually Morishima's secretary mainly) to see when Michio was free. One would refer to Professor Morsihima by his first name but I could never bring myself to call him so; his personality (and maybe my background) was

so powerful, I felt that I could never bring myself to use the first name in conversation in his presence. Also as a part of my duties as lecturer, I was required to supervise several undergraduates, discuss their progress, get them to write essays and help them through what was really quite a demanding programme. I recall, several of them were actually taking their first course in Economics, taught by Professor Morishima and the lectures were based on his book **Economics for a Modern Society**, Oxford University Press, 1973. I had not seen the book and quickly spent a few days in reading it.

I realized that Professor Morishima by then had developed second thoughts about the use of mathematics; in fact, in confirmation, when I gave him a copy of my paper which I had been working on then, he said that he had lost interest in such matters and I did point out that I had been attracted to study economics by his own attempts at the use of mathematics. In any case, I remember that I asked my students whether they would be comfortable with mathematics and since there were no problems on that score, I rewrote several portions of the book using mathematics and used them for discussions. I remember that my class size grew since people found that following my math was much simpler than following Professor Morishima's apparently 'non-mathematical' arguments¹⁸.

Prior to this change of interests, we had been introduced to his famous book **Equilibrium Stability and Growth**. I started reading this book when I was a graduate student in the late sixties and early seventies. In some sense, Morishima had set Hicks as his role model and therefore, his Equilib-

¹⁸Professor Morishima often described his command over the English language as being less than perfect; he was not particularly modest on this count since he described his English as being 93% English.

rium Stability and Growth should perhaps be seen as a follow up on Hicks' own Value and Capital. In fact, in a later book¹⁹, Morishima acknowledges the effect that Hicks had on him, when he talks of his experiences during the War when he was a gunner and took solace from reading Hicks' Value and Capital. The war obviously did not affect Morishima's judgement on academic matters.

Professor Morishima's interest was not merely on equilibrium but also on its stability and on the problem of growth. He was greatly influenced by the classical contribution of von-Neumann (1937) on the model of an expanding economy. He tried to mould this, to the Walrasian tradition, and provided successive books to interpret the contributions of Ricardo, Marx, Cassel and Hicks within this framework. These names would reveal that Morishima was quite focused on the issues relating to growth and development.

One of his interests was in studying linear structures. In a sense, he differed from his contemporaries on this matter: the production structure he used was almost always linear. This preference for the linear structure, perhaps rose for his deep commitment to the notion of economics being essentially dynamic and for dynamic considerations linear structures were helpful. This could have been also due to the fact that he was at heart a classical scholar and for him, constant returns to scale was the ideal structure and this naturally imposed the linear structure. In his work he tried to generalize by getting rid of artificial assumptions²⁰. He also studied generalizations of

¹⁹**Theory of Economic Growth**, Oxford University Press, 1969. In fact in the preface, Morishima talks of the "exciting experience of running a race with the champion" referring to the fact that Hicks was writing **Capital and Growth** at the same time and place as he was when he began work on this book.

²⁰Consider for example, his pithy comment on additivity so routinely assumed: A process

properties of non-negative square matrices and proved a turnpike theorem for the no-joint production case. To provide a sample of his very diverse contributions and do justice would be a very difficult task.

Consider for example his contributions to the theory of stability of equilibrium: one of the principal conditions for stability of equilibrium was the assumption that goods be gross substitutes of one another. He realized that this was an overly strong requirement and devoted much time in introducing complementary goods in a way that it would not affect the stability of the system. This led to the development of what came to be known as a Morishima Matrix; global stability consideration in such frameworks continue to be analyzed even relatively recently²¹. Given the influence of Hicks on Morishima, a fact we have mentioned above, it is not surprising that Morishima also turned his hand on enlarging the scope of Hicksian Laws of Comparative Statics²². There is another aspect that he attempted, that of attempting to talk about not only small changes but arbitrary changes or shifts and he showed how he could use some special structure of gross substitute systems to achieve this end.

Writing more recently²³, Morishima explicitly recognizes the dynamic nature of economics; his point of departure lies in the choice of given production sets for producers; this according to him denies the very role of producers as

producing rabbits and one producing airplanes can never be amalgamated efficiently. “On Two Theorems of Growth Economics: A Mathematical Exercise”, *Econometrica*, 1965.

²¹D. C. Keenan, Morishima Systems and Global Stability, *International Economic Review*, 1990.

²²See for instance, L. Basset, H. Habibagahi and J. Quirk, Qualitative Economics and Morishima Matrices, *Econometrica*, 1967.

²³*Capital and Credit: A New Formulation of General Equilibrium Theory*, Cambridge University press, 1992.

entrepreneurs and he defines entrepreneurial activities as those creating new production sets. In one sense, of course he borrows from Hicks and that is in the notion of the Hicksian week; and his producers need access to credit and capital so that they can buy inputs now to produce and sell outputs later, at the end of the Hicksian week. *Capital and Credit* was written to set right the deficiencies he pointed out. In fact from the early 60's Morishima had been trying to achieve a consistent story which would satisfy him and in *Capital and Credit*, he comes close to saying that he has arrived (although, he does leave room for improvement by adding the word 'temporarily'). A later book²⁴ is actually made up of two parts: the first part is an English translation of Morishima's early essay written in Japan in the 1950's. It is interesting to note that this contribution was made with a view to make the model of Value and Capital more general by including inventories into firm behavior and by explicitly studying the role of financial assets and trading into firm behavior: an aspect which seem to have occupied him from the beginning to the end. He also formulates stability analysis for not only temporary equilibria but also the time path of temporary equilibria. All this was done in the 1950's. A part added after the translation is a piece on why the stability analysis is different from the literature; there are also appendices which determine the Keynesian liquidity preference schedule in a proper general equilibrium context.

Apart from theoretical explorations into dynamics, he also wrote at length on the Japanese economy detailing not only why Japan had succeeded²⁵; in one of his last books, he detailed why he thought that Japanese economy

²⁴**Dynamic Economic Theory**, Cambridge University press, 1996.

²⁵**Why has Japan "Succeeded"?** **Western Technology and Japanese Ethos**, Cambridge University Press, 1982.

would collapse; this was in 1999 and turned out to be prophetic in many ways²⁶. It should be noted that Morishima puts Japan's success within quotation marks signifying perhaps that he did not think much of it; be that as it may, he attributes this so-called success to Japanese Confucianism which supported the values of self-sacrifice and loyalty leading to the development of loyal and very hardworking workforce which aided the rapid postwar growth. Morishima also notes that this rapid growth was also accompanied with 'extreme nationalism, paternalism and anti-individualism'. These attributes are not considered favourably by Morishima and it is because of this maybe that in the latter book he predicts that Japan will become a third-class economic power by 2050. Naturally these dire predictions were hotly debated inside Japan and since the book was written in Japanese, it did not make him too popular in Japan.

Finally, I can do no better than reproduce what another distinguished dynamic theorist Richard Day²⁷ had to say about Morishima's work: "Morishima plumbed the depths of Hicks and Keynes, clarified their insights, discovered the points at which they stumbled, ingeniously extended their analysis in a way which enables us to understand their achievements. At the same time he reveals the fundamental theoretical problem at the roots of their static methods....Along with other great theorists of his generations, he stands as an inspiration....partly for what he has done even more for what he tried to do."

Any assessment of Morishima will be incomplete if we did not note his

²⁶**Naze Nihon Wa Botsuraku Suruka?** *Why will Japan Collapse?*, Iwanami Shoten, 1999

²⁷Review of Dynamic Economic Theory, **Journal of Economic Literature**, Vol. 36, 1998, 939-941.

success in building institutions. In such activities, Professor Morishima has left his mark in many ways. I suppose one must begin with his efforts together with other Japanese stalwarts which resulted in the building of the Institute of Social and Economic Research (ISER) at Osaka University. Any scholar who has had the privilege of spending time there can attest to the brilliant idea of having a few full time scholars and two visitors in a small but extremely well organized institution. During this phase of his career, he was also instrumental in setting up the journal, *International Economic Review*, together with Lawrence Klein at the University of Pennsylvania; even to this day, the journal is published jointly by these fine institutions and is known as one of the top journals of Economics. I was also privileged to have been at the LSE when Professor Morishima was organizing the huge funds under the Suntory-Toyota Foundation to set up STICERD (Suntory and Toyota International Centre for Economic and Related Disciplines) at the LSE; this centre has surely helped LSE maintain its position as a premier institution engaged in teaching and research in economics.

Not only the institutions mentioned above, but Japanese educational policy was influenced by Professor Morishima. I am informed that Professor Morishima also persuaded Prime Minister Fukuda of Japan in the 1970's to change the national university regulations so that it became possible to employ foreigners as "regular" tenured professors²⁸.

²⁸It may be of some interest to note that Dr. Ravi Ranade, who completed his Ph.D. at the Centre for Economic Studies and Planning, JNU under the supervision of Professor Satish Jain was the third foreigner employed under this new system. The preceding two foreigners had in fact already been in Japan when appointed, and so Dr. Ranade was the truly first "foreign" teacher who got a regular post in a National University of Japan, thanks to the efforts of Professor Morishima.

No matter how one looks at his contributions, whether from a purely academic side or from the point of view of his contribution to institution building, it is clear that Michio Morishima set standards which are impossible to come close to.